Comparison of Mean Field Annealing and Multiresolution Analysis in Missing Data Estimation

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Abstract

The project we are working on is to help develop and test a low cost, large area, high resolution X-ray detection system with a high dynamic range. The large area is achieved by butting two or more scintillator/fiber/CCD combinations together. An algorithm is thus required to compensate the defects come from the detector induced errors including missing single pixel due to individual defective detectors or missing column(s)/row(s) due to misalignment of adjacent CCD's in the blurred and noise corrupted images. Mean field annealing, as a global optimization technique, is the proposed algorithm. This paper, however, proposes a new approach based on multiresolution analysis where the defect compensation is implemented by removing the characteristics created by the missing columns/rows from the *detail images* of lower resolution. Experiments will be carried out to compare the performance of these two approaches. Future research directions are discussed at last.

1 Introduction

The requirement for a missing data estimation algorithm is originated by the project where two or more scintillator/fiber/CCD will be butted together to implement a low cost, large area, high resolution X-ray detection system with a high dynamic range. Here, fiber optics is used to develop the light-guide system such that light from an X-ray scintillator may be guided down to a CCD array preserving the alignment of all the pixels in a 1100 x 1200 array. Though defect free devices are theoretically available, they are much more expensive than devices with only a few defects. Thus an algorithm is required to compensate the defects and misalignment, allowing much lower cost detectors to be used, thereby reducing the overall cost of the system.

Mean field annealing (MFA) is the proposed algorithm for this project. It combines annealing technique and mean field approximation to find the global minimum. By minimizing an objective function according to different problem definitions, MFA can achieve noise removal with sharp edges preserved [1], restoration of locally-homogeneous [4] images, optimal image interpolation of missing data [8], etc. As we can see in Sec. 5, MFA can obtain good results in missing data estimation as well.

Multiresolution analysis (MRA), on the other hand, is proposed here in order to compare with the performance of MFA. Since its formulation in 1986, MRA has found its applications in many areas of computer vision, such as image compression, edge detection, texture analysis, image restoration, etc. Having a stable mathematical foundation from wavelet theory [2][6][7], MRA provides us a new tool to analyze image contents. In this paper, we try to implement the missing data estimation by analyzing the detail images of lower resolution, removing the characteristics that generated by the missing columns/rows, and estimate the missing data through reconstruction process.

The organization of this paper is as follows: after problem definition in Sec. 2, we introduce the MFA approach in Sec. 3, and MRA approach in Sec. 4; experiments and results will be presented in Sec. 5; conclusions and future research directions are discussed in Sec. 6.

2 Problem Definition

The project we are working on is to help develop and test a low cost, large area, high resolution X-ray detection system with a high dynamic range. The large area is achieved by butting two or more scintillator/fiber/CCD combinations together (Fig. 1). In developing such a kind of system, the trade-off between cost and image quality must be considered. Defect free devices are much more expensive than devices with only a few defects. Therefore, an algorithm is required to compensate such defect so to reduce the cost of the overall system.

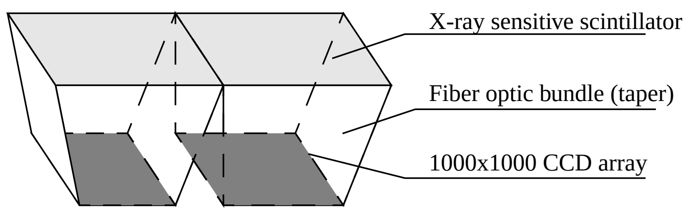


Figure 1. Two scintillator/fiber/CCD combinations may be butted together.

The defect of the image mainly come from three sources: 1) *detector induced errors* including missing single pixel due to individual defective detectors or missing column(s) or row(s) due to misalignment of adjacent CCD's; 2) *blur* caused by "point source" x-ray systems; and 3) *spatially-varying noise*.

$$g_i = (f \otimes H)_i + n_i + d_i \qquad (1)$$

Given the corrupted image g, generated from Eq. (1), the proposed algorithm should be able to find the estimated $\hat{f}$ which is a perfectly-aligned, defect-free detector based on the information of blur, noise, and detector induced errors. The whole process is like the model in Fig. 2.

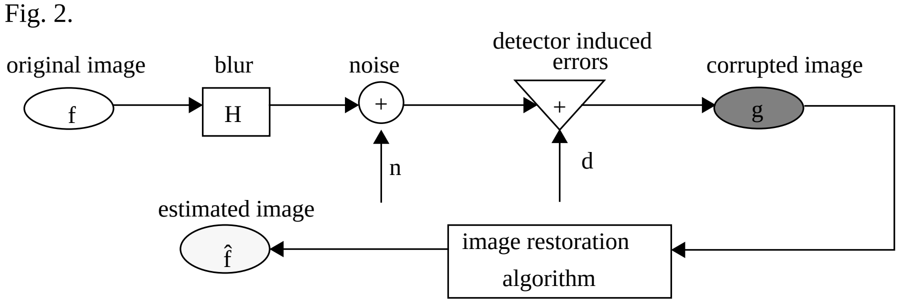


Figure 2. A model for image corruption and reconstruction process.

3 Mean Field Annealing (MFA) Approach

MFA is the proposed algorithm in our project because it has good performance in both noise removal and image interpolation. Though the determination of the missing data is (in general) impossible, MFA is still optimal in its ability to estimate those values, given the suitability of the blur, noise, and detector induced errors. Even in the case that the blur is not accurately modeled, the prior term allows edge-preserving interpolation very well.

The problem defined in Sec. 2 can be modified into a global optimization alternative, i.e. to seek image f, which will maximize the *a-posteriori* conditional probability $P(f|g)$. To solve this problem, Bayes' rule is used to derive Eq. (2),

$$Max(P(f|g)) = Max\left(\frac{P(g|f)P(f)}{P(g)}\right) \tag{2}$$

where the denominator $P(g)$ is independent of f and therefore does not affect the maximization process. We denote the conditional probability $P(g|f)$, which depends on the corruption process, as the *noise term*, and the *a-priori* probability $P(f)$ which is independent of observed image, as the *prior term*.

By taking the logarithm operation on Eq. (2), one can get the objective function as Eq. (3),

$$H = H_n + H_p \tag{3}$$

where,

$$H_n = \sum_{i,j} \frac{(f_{i,j} - g_{i,j})^2}{2\sigma^2} \tag{4}$$

$$H_p = -\sum_{i,j} \left(\frac{\beta}{T}\right) exp\left(-\frac{(\nabla f)^2_{i,j}}{2T^2}\right) \tag{5}$$

The algorithm carries out the minimization by combining conventional gradient descent (Eq. (6)) with *annealing*. The annealing minimization process starts with large T and gradually reduces the temperature over time. This process avoids most local minima and produces an optimal restored image.

$$f^{k+1}_{i,j} = f^k_{i,j} - \alpha \frac{\partial H}{\partial f_{i,}} \tag{6}$$

In the experiments of Sec. 5, given g corrupted by zero mean Gaussian noise and one missing columns due to the misalignment of the adjacent detector, we first apply a 3x3 median filter on g and a special 5x5 median filter around the missing columns to obtain the initial estimation of image f. Then based on Eqs. (4) ~ (6) with gradually reduced temperature, the final optimal estimated image of f will be worked out.

4 Multiresolution Analysis (MRA) Approach

Multiresolution analysis, since its formulation in the fall of 1986 by Mallat and Meyer, has been found important in analyzing the content of images. Natural applications of MRA are recognized as the image compression, edge detection and texture analysis. Recently, researchers found some more applications and got good results, such as image restoration and noise removal.

MRA tries to understand the content of the image at different resolutions, because the details of an image generally characterize different physical structures of the scene.

The 2-D MRA first convolves the rows with a one-dimensional filter, retain every other row, then convolves the columns of the resulting signals with another one-dimensional filter and retain every other column. The framework is drawn as Fig. 3 where $\phi(x)$ is the scaling function (like a low-pass filter) and $\psi(x)$ is the wavelet function (like a high-pass filter.

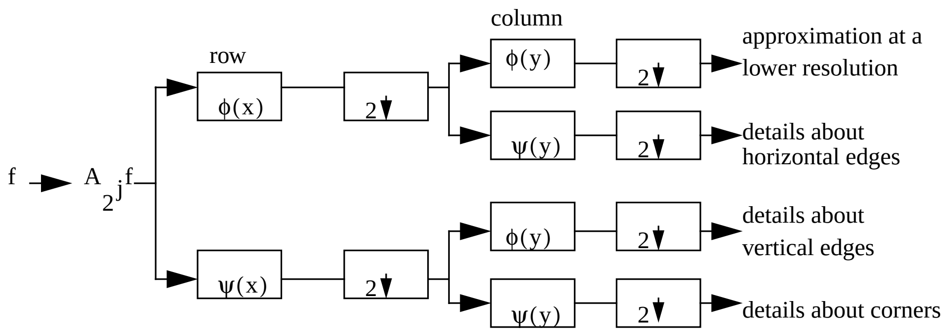


Figure 3. Decompose an image from a higher resolution to a lower resolution.

The application of wavelets in the area of image interpolation is not as popular as that in denoising, especially for missing data estimation. Only few papers try this area. Chang *et. al.* [3] is one of them where they propose a wavelet based method which estimates the higher resolution information needed to sharpen the image, i.e. it can extrapolate the wavelet transform of the higher resolution based on the evolution of the wavelet transform extrema across the scales.

For the missing data estimation problem, we propose the multiresolution decomposition based method, where in each resolution, the sharp edges caused by the missing data is detected and deleted, the information missed there is then interpolated (estimated) based on the vicinity of each pixel. On the other hand, since the details in the original image will disappear while the resolution goes smaller and smaller, theoretically, we can reconstruct the optimal image by using the approximated image in the smallest resolution and the processed detail images in the same resolution to obtain the approximation image in the adjacent larger resolution, and repeat this process until we get the final optimal image which is in the same resolution as the original image (Fig. 4). Though theoretically correct, we can't reduce the image resolution to infinity, thus some defect might occur depends upon the property of the original image.

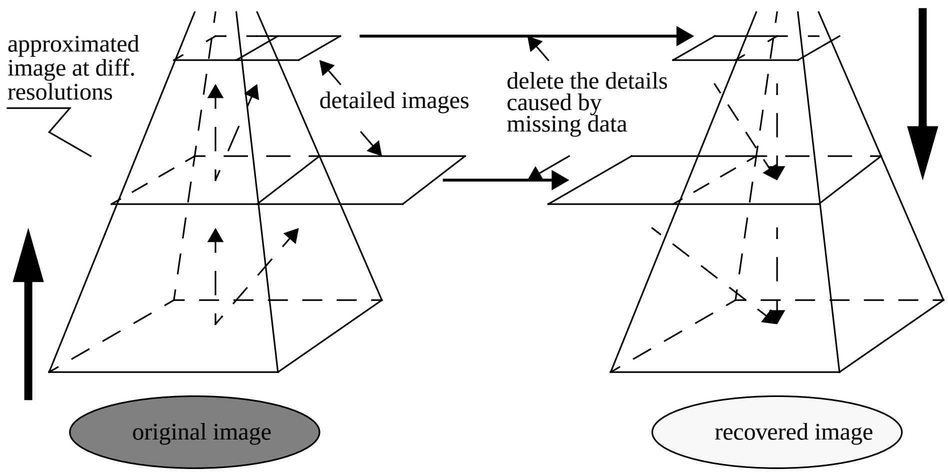
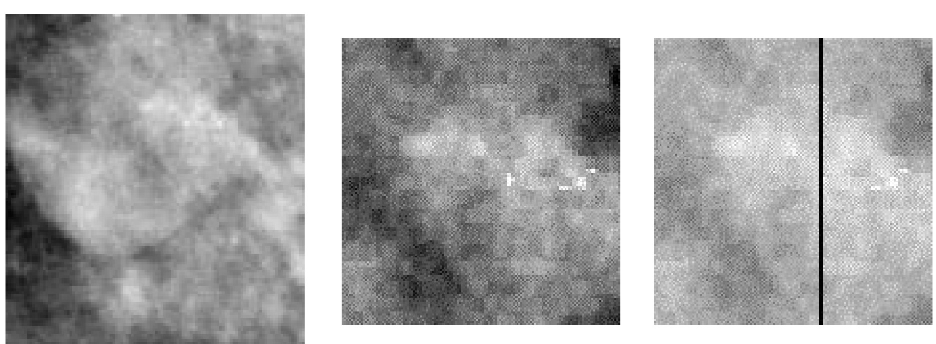


Figure 4. Model of MRA approach in using wavelets to solve missing data estimation problem.

5 Experiments and Results

We choose mammogram to do our experiments (Fig. 5a). This image is scanned at 100 micron (5 linepairs per mm) resolution. Fig. 5b is a segment (64x64) of Fig. 5a, where two associated pleomorphic microcalcifications are apparent. Fig. 5c is the same image as Fig. 5b but with one column (just the position where one of the microcalcification locates) missing. Five methods are used to recover the missing column and the results are shown in Fig. 6. Fig. 7 is the decomposition of the missing column image with db2 as the mother wavelet. The vertical edge caused by the missing column is clear.



(a) (b) (c)

Figure 5. (a) original image; (b) testing image; (c) testing image with one column missing.

We have mentioned two recovering methods in previous sections (MFA and MRA). The three others are much simpler.

- *random copy method*: randomly copy the value from the left or right neighbor of the missing pixel;
- *average substitution method*: use the average value of the left and right neighbors as the estimation of the missing pixel; and
- *median substitution* method: use the median value of the local neighbors as the estimation of the missing pixel.

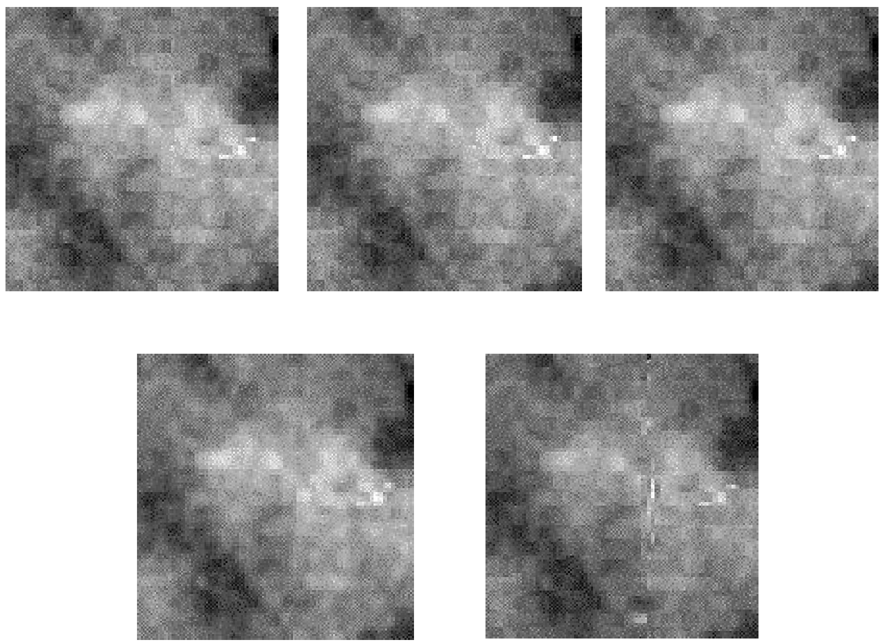


Figure 6. Recovered image. (from upperleft to lower bottom: by random copy, by average substitution, by median substitution, by MFA and by MRA).

From Fig. 6, we can see that the upper three images didn't recover the missed microcalcification at all, but the bottom two can both recover it to some extent. On the other hand, result from MFA is more blurred than that from MRA, and MRA also generates some apparent artifacts the same time it preserves the characteristics. The oversmoothing from MFA mainly comes from the contribution of prior term where a globally smooth image is supposed to be reached.

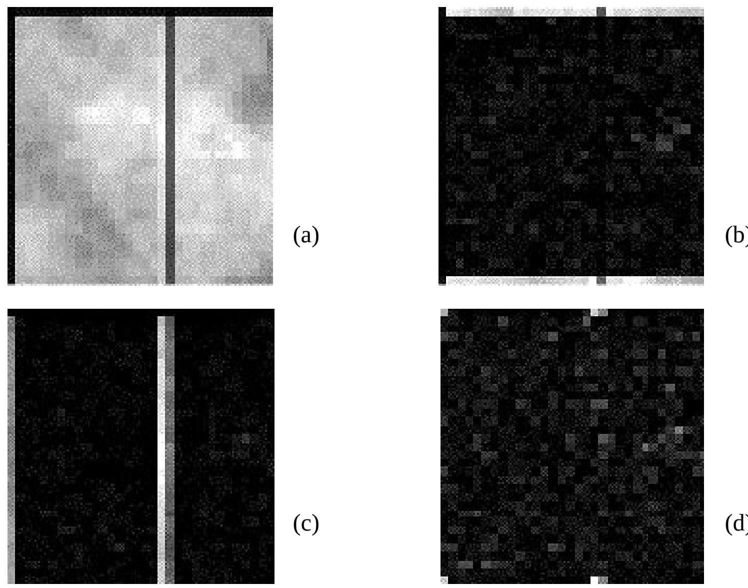


Figure 7. Decomposition of Fig. 5c with db2 as the mother wavelet. (a) approximation at lower resolution, (b) horizontal detail, (c) vertical detail, (d) corner detail.

6 Future work

Though both MFA and MRA can recover some important characteristics from the missing column images, they both, however, have some disadvantages, such as MRA's artifacts and MFA's oversmoothing. Recently, we are trying to use the blur information to recover the missing column exactly or at least with great accuracy. This method can recover the missed information totally if the blur kernel is separable and exactly known. However, when the image size goes larger and larger, the recovered image turns very unstable. How to solve this ill-conditioning problem is the next research topic.

7 Reference

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